SOCIETY OF PHYSICS STUDENTS
An organization of the American Institute of Physics

# SPS Chapter Research Award Proposal

| | |
|---|---|
| **Project Proposal Title** | Simulations of Black Hole Dynamics: From Event Horizon to AMD Ryzen |
| **Name of School** | University of Central Florida |
| **SPS Chapter Number** | 1076 |
| **Total Amount Requested** | $2,000.00 |

## Abstract

SPS at the University of Central Florida plans to accurately simulate particle motion near black holes using a student-built computer. This work will introduce our members to scientific computing, produce free and open-source simulation software, and help develop an understanding of basic concepts of General Relativity.

# Proposal Statement

## Overview of Proposed Project

Black holes are interesting physical objects, the existence of which is predicted by General Relativity. They have been important in recent years, among other reasons, because the gravitational waves famously observed at LIGO were produced in a black hole collision process. These objects traditionally arise in theory as special classes of exact solutions to the Einstein field equations, a set of nonlinear partial differential equations determining the gravitational field. Because understanding black holes in this way requires somewhat sophisticated techniques, the physics behind them is sometimes erroneously assumed to be esoteric and is not fully appreciated, in any quantitative sense, by wider audiences.

In this research project, we propose to provide undergraduate students at the University of Central Florida with an opportunity to understand some aspects of black hole physics, and more broadly the problem of relativistic orbits, using basic Classical Mechanics as well as computer simulation. Specifically, the team of students will build a computer which will run simulations of particle motion in an Einsteinian gravitational field by numerically integrating the equations of motion in various circumstances. In simple cases, such as the Schwarzschild black hole, it is possible to integrate the equations analytically using techniques of Classical Mechanics and the results may be compared. Even in these basic cases, one can already observe new physics, such as the failure of orbits to close and the existence of an event horizon; the computer simulation will give students the means to visualize this in a very concrete fashion. This will allow students to further develop their skills with Classical Mechanics, scientific computing, and build some basic intuition for General Relativity without confronting too many technical details.

This project will strengthen the objectives of the SPS program by exposing a wider audience to an exciting piece of physics. When the project is completed, we plan to make the simulation software freely available online, so that an arbitrarily large audience of people can begin to interact with the physics.

## Background for Proposed Project

A historic problem in physics, which is widely taught in Classical Mechanics courses, is the Kepler problem dealing with motion in a spherically symmetric potential varying as $1/r$ . After appealing to standard arguments involving conservation of energy and angular momentum, the Kepler problem can be reduced to an effective one-dimensional problem for the radial coordinate with effective potential (in units where $G = c = 1$ ):

$$V(r) = -\frac{M}{r} + \frac{L^2}{2r^2}$$

where $M$ is the mass of the object being orbited and $L$ is the angular momentum of the orbiting test particle. From this effective potential, the motion can be solved by quadrature. From this solution, one reads off standard properties such as the period of the motion, the fact that the trajectories are conic sections, and so on (see, for example [2]).

In Einstein gravity, the simplest black hole geometry is the so-called Schwarzschild metric, corresponding to line element

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (1 - \frac{2M}{r})dt^2 - (1 - \frac{2M}{r})^{-1}dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2 .$$

This describes the gravitational field of an uncharged, non-rotating black hole of mass $M$ . The trajectories of test particles correspond to geodesic motion in this geometry. Due to the high degree of symmetry in the line element (time translation symmetry as well as rotations), there are several conserved quantities which make the relativistic case integrable, just as the standard Kepler problem. After the analysis has been completed, the end result is again an effective one-dimensional problem for the radial coordinate, but now in the effective potential (for details, see for example [3]):

$$V(r) \;=\; -\frac{M}{r} + \frac{L^2}{2r^2} - \frac{ML^2}{r^3}.$$

At this point, the motion can be analyzed just as before, but now with this expression for the potential. The third term is interpreted as a relativistic correction, and restoring units one sees that it is suppressed by a power of $r_S/r$ relative to the other terms, where $r_S$ is the Schwarzschild radius of the object in question. The period of the orbit is now given by an elliptic integral and it's possible to analyze small corrections to Newtonian results, some of which reveal new features such as the failure of orbits to close. Most interesting for black hole physics is that one can already see that for trajectories of sufficiently high energy directed radially inward, the Schwarzschild radius is a "point of no return" and all matter and radiation become trapped in that region. This is the hallmark of a black hole.

What we propose to have students do is to run simulations of particle motion in this and other geometries by numerically integrating the equations of motion which allows them to concretely visualize the trajectories on the computer screen. For previous work numerically integrating such equations of motion, see [1]. In simple cases such as that outlined above, this amounts to a slight variation on familiar themes from Classical Mechanics which reveal new physics. However, it's possible to explore wide classes of geometries with this synthesis of analytical and numerical methods. More complicated black hole solutions, such as Reissner-Nordstrom, Kerr, and Kerr-Newman, may also be analyzed, and with the assistance of the software students may even analyze geometries where it is impossible to solve the equations of motion analytically.

## Expected Results

We plan to produce a collection of simulations of particle motion near black holes with varied spacetime geometries and initial conditions. We will also properly document our code and make it available on a public-facing site, such as GitHub. We will also make the simulations available on our SPS chapter website. This project will expose the participants to General Relativity, Mechanics, Python programming, and computer hardware. Much of modern physics research involves writing code, and this project will prepare our members for such research. Our members will also learn how to build a computer, set up a reproducible computing environment, and work on a multi-user system. This is useful because most computational physics research is done in an environment much like the one we will set up.

By making our results publicly available, we will expose a large group of people to a topic that is usually out of reach for those who do not actively study physics.

## Description of Proposed Research - Methods, Design, and Procedures

We plan to begin by reviewing the relevant aspects of Classical Mechanics for this project, focusing in detail on conservation laws and the Kepler problem. We will then provide a brief overview of the relevant background in general relativity needed to understand the origin of the equations of motion that we will be solving, so that participants understand the motivation for the project.

We will supplement this with an introduction to numerical computing, introducing members to Python, Linux, and computer hardware. This will include an overview of standard data analysis libraries such as NumPy and SciPy and also more advanced libraries such as Dask for parallel computing. To assess members' knowledge, and as a warm-up before working on the actual project, we will have them run a simulation of the Kepler orbit problem considered previously so that they can see the concepts they have learned so far displayed.

When we receive the first $1000 of funding, we plan to begin building the computer with members using the available parts. We will then set up the computing environment and help our members get comfortable with working on the system. As a warm-up, we will then have our participants write code for simulating motion in the Schwarzschild black hole, run it, and validate it. Since the solution in this case is known analytically, and may also be done using the simpler numerical methods they have practiced with their laptops, it will help them understand how the computer is working and prepare them to generalize to more complicated situations.

When the second $1000 arrives, we plan to make upgrades to the existing computer so that it can do more strenuous computations. This will prepare members to simulate more complicated situations, involving the Kerr, Reissner-Nordstrom, and Kerr-Newman solutions. We anticipate that some of these may present complications that require more sophisticated numerical solvers.

We plan to adhere to FOSS software standards throughout the duration of this project, and publish the code routinely on a site such as GitHub. This will help teach our participants about reproducible research and the process of licensing code. When the project is completed, we plan to make the results and code freely available to the public on our chapter's SPS website.

## Plan for Carrying Out Proposed Project

We plan to assemble a team of solely SPS members but are open to outside participation. Other than the two SPS officers leading the project, we currently have a group of six SPS members who are committed to the project. These six members will be UCF students for the duration of the project and will act as facilitators to involve the entire chapter at UCF.

The project leaders, David Wright and Spencer Tamagni, pool together an extensive knowledge of mechanics, general relativity, scientific computing, and computer hardware, coming from their coursework (including graduate classes) as well as research projects. Spencer does research in mathematical physics under the supervision of our chapter's SPS advisor, Prof. Costas Efthimiou. His research experience and coursework qualify him to lead the aspects of this project involving theoretical physics. He has taken a graduate class in General Relativity and his research work deals extensively with the use of differential and algebraic geometry in physics (differential geometry is the mathematical language required to do General Relativity). He has, with Prof. Efthimiou, written a paper explaining how to use geometric techniques in the context of electrostatics, and is working on a book detailing geometric aspects of exact results in quantum field theory. David Wright brings years of research experience in computational astrophysics to the table. He is currently developing a software package that will accurately simulate observations of planetary transits made by the upcoming James Webb Space Telescope. In addition to research, David is also the system administrator for his research group's computing cluster. This position requires extensive knowledge of computer hardware, Linux, Python, and a myriad of other tools related to computational science. The combination of these two positions qualify David to lead the computational aspect of this project.

With COVID, research space is uncertain. We plan to work with the UCF Physics Department to secure a room for building the computer. We will then put the computer in the UCF physics department's server room. All of the work other than building the computer can be done remotely; in particular we do not expect COVID to severely hamper the development of this project.

## Project Timeline

**January 2021** - Receive funds, finalize parts list, order parts, begin reviewing background with members.

**February 2021** - Begin receiving parts and building computer. Continue reviewing background with members. Apply to present at UCF Showcase of Undergraduate Research Excellence (SURE).
**March 2021** - Finalize computer build. Set up a computing environment and teach members how to work on the computer. Finish background review with members.
**April 2021** - Begin work on simulating particles near Scharwzschild black hole. Present current results and future plans at SURE, possibly attract new collaborators to the project.
**May 2021** - Finish Schwarzschild simulations. Write interim report.
**Summer 2021** - Receive second half of award. Upgrade computer with parts needed to run more strenuous simulations. Begin work on Kerr black hole simulations.
**September 2021** - Finalize Kerr and begin Reissner–Nordström black hole simulations.
**October 2021** - Finish Reissner–Nordström and begin Kerr-Newman simulations.
**November 2021** - Finish Kerr-Newman work and begin writing final report. If code and results are not already available to the public, make them so.
**December 2021** - Complete final report, submit to SPS. Apply for funds to send a member to a conference and present our work.

## Budget Justification

In the attached budget, we have listed all of the parts we will need to build a high-performance computer that will enable us to support multiple users and run simulations in a timely manner. The most expensive item in our budget is the processor for the computer. We need a high core-count processor to support the multiple users and workload we are anticipating, and many of the calculations involved benefit from parallelization which would not be feasible on an average computer. The rest of the parts are standard for a machine oriented towards computation.

We have also included a handful of books which the project leaders will use to help teach the background knowledge the members need. The books will also be made available to members as a reference.

Due to the two-part nature of the award, we will spread out the purchases. In the first round of funding, we will buy only the necessities to get our computer up and running. We will purchase the additional materials when the rest of the funding is received. We have also considered asking the Physics Department at UCF to match or partially match the initial funds and then use the second round of funding to reimburse the department.

We plan to host the computer in the UCF Physics Department's server room. We will utilize the monitors, rack space, and high-speed internet connections available.

## Bibliography

[1] F. Bacchini, B. Ripperda, O. Porth, L. Sironi "Generalized, energy-conserving numerical simulations of particles in general relativity. II. Test particles in electromagnetic fields and GRMHD", *Astrophys. J. Suppl.* 240 (2019) 2, 40.

[2] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics, Volume I: Mechanics.* Elsevier Science, 1976.

[3] R. M. Wald, *General Relativity.* Chicago Univ. Press, Chicago, IL, 1984.