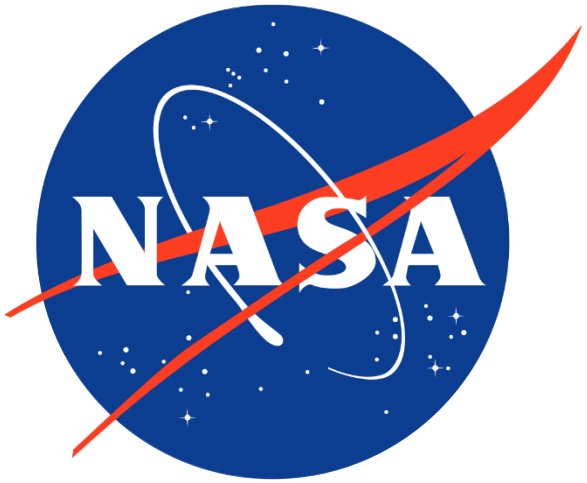


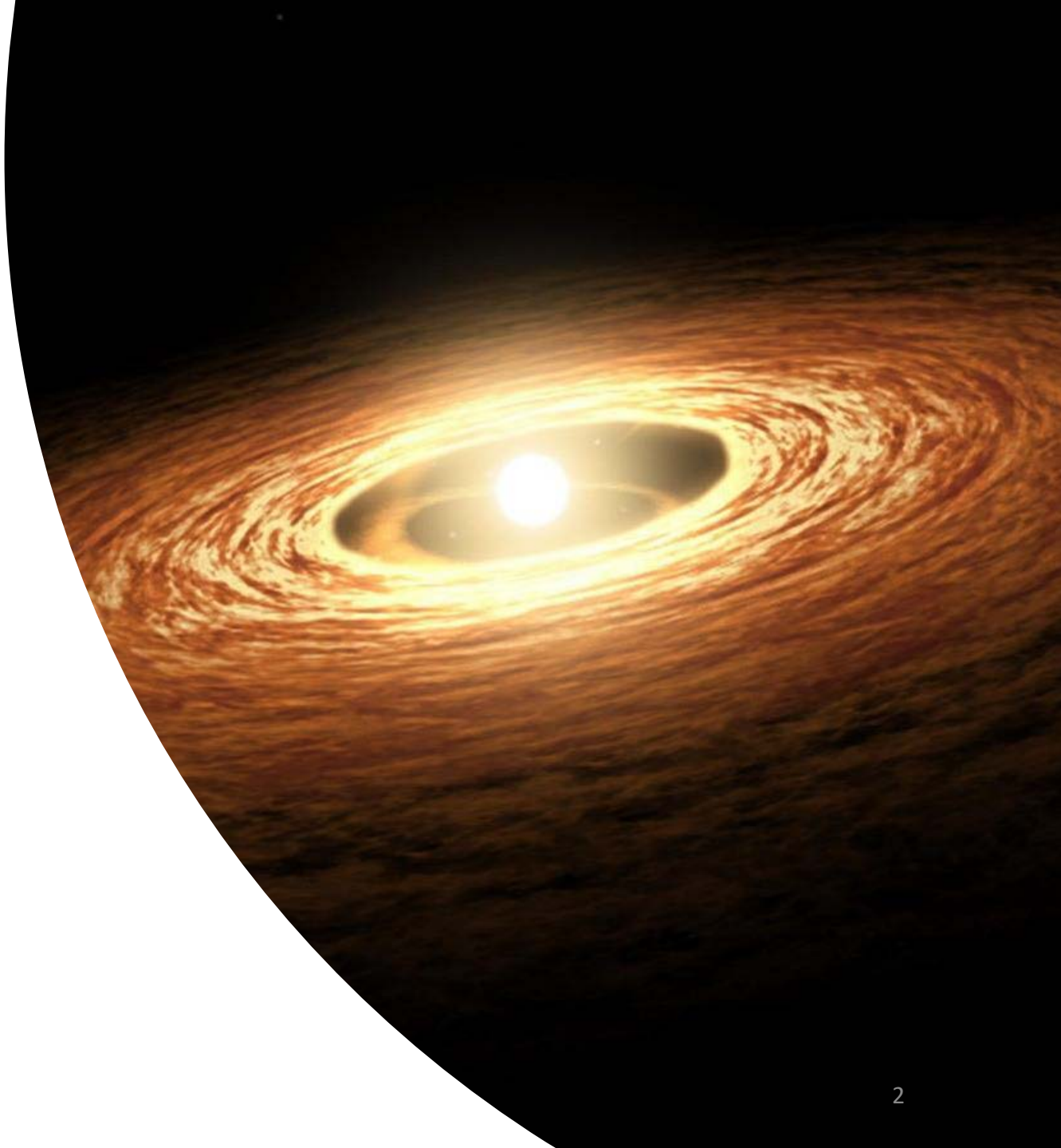
Evaluating bi-orthogonality of Rayleigh-Lamb eigenmodes in elastic plates

Terry Schuh

Mentors: Karwan Rostem and Edward Wollack

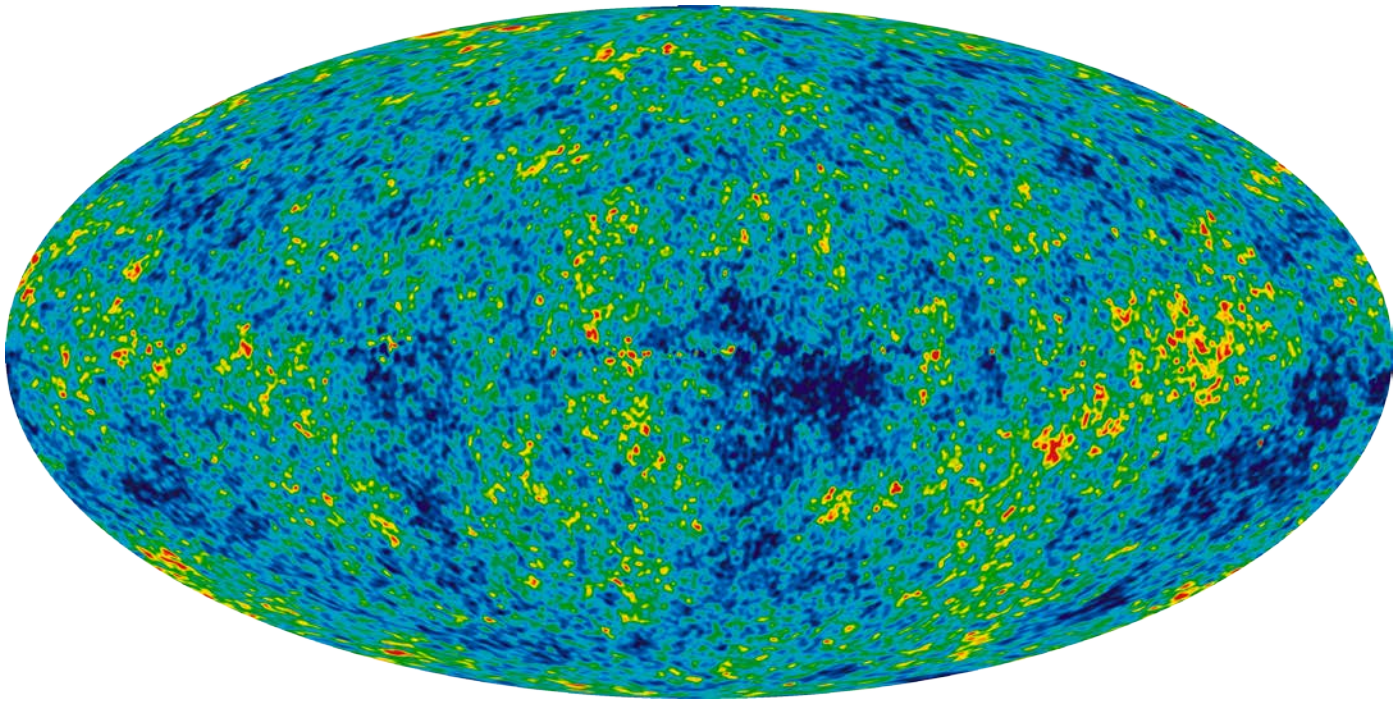
Motivation

- Astrophysics requires more and more precise measurements
- This requires extremely sensitive detectors
- To reach desired sensitivities, detectors need to be very cold and, components need to be very small in size



Very Cold

$$T_{detector} \ll T_{source}$$



$$T_{CMB} \sim 3\ K$$
$$T_{detectors} \sim 0.1\ K$$

Very Small

E-M Spectrum

Increasing Frequency (ν)

γ rays	x rays	UV	visible	IR	Microwaves	Radio

Increasing Wavelength (λ)

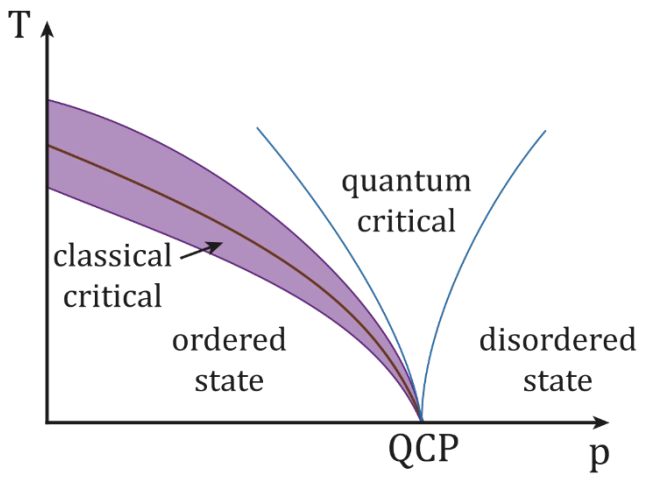
observational cosmology lab

λ ranges from 0.1 – 10 mm

Scale of detector physical features are ~0.5 – 1000 µm

Mesoscopic Limit

- At these temperatures and physical scales, device operation approaches the mesoscopic limit
- Temperatures < 1 K
- Electromagnetic wavelengths set the ~1 mm pixel scale
- Thermal radiation wavelengths set scales of detector dimensions
- Classical system → quantum system
- Continuum limit → discrete modes

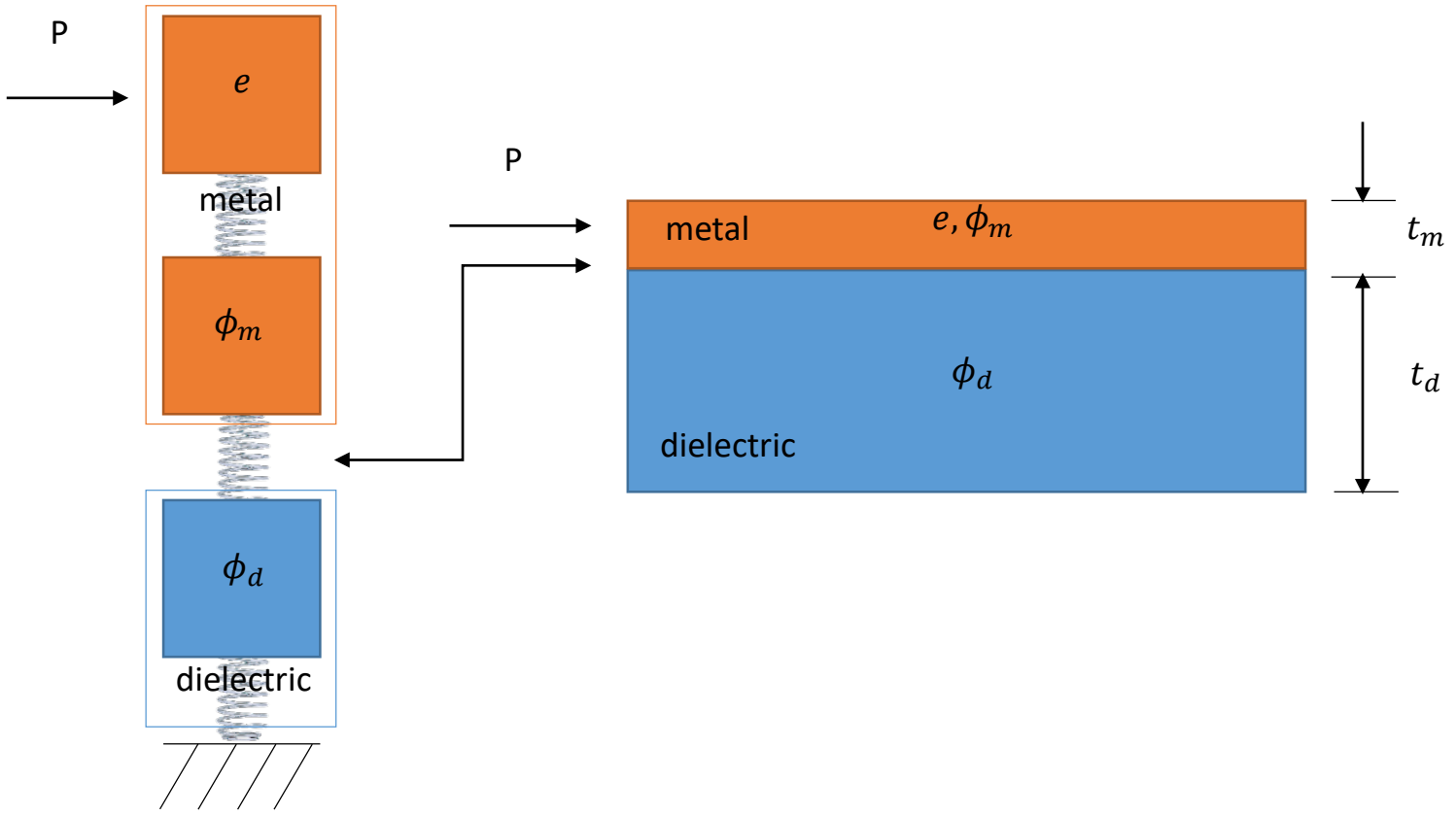


Our Goal

- Metals and dielectrics are integral to radiation detectors
- Electron-phonon (e-ϕ) interaction is key to detector response
- Need to understand this interaction to make detectors work
- That's what I worked on!

How To Do It

- At the mesoscopic limit, phonons treated as elastic waves in a solid
- Thus we evaluate the phonon modes and use “bi-orthogonality” to understand coupling



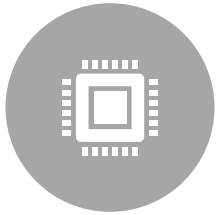
Conclusions and Future Work

- Wanted to improve sensitivity of current astrophysical detectors
- Important to understand e-ϕ and ϕ-ϕ coupling at low temperatures and in nano-scale structures
- Successfully evaluated bi-orthogonality relation between elastic eigenmodes in uniform/single layer silicon plate

- Model more complicated, multi-layered plate using COMSOL
- Evaluate bi-orthogonality relation for eigenmodes in complete model

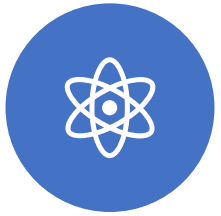
What I Did

Worked with bi-orthogonality



Used COMSOL and MATLAB everyday

Developed and solved a simplified version of plate



Learned a ton more physics

Improved coding abilities

Developed more research skills

Acknowledgements

- SPS organization
- NASA mentors Karwan and Ed
- Lab group → Kyle, Katherine, Jake, etc.
- Fellow SPS interns

NASA
Goddard Space Flight Cent

Bonus Slides

References

- Fraser, W. B. "Orthogonality relation for the Rayleigh–Lamb modes of vibration of a plate." *The Journal of the Acoustical Society of America* 59.1 (1976): 215-216.
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- Gunawan, Arief, and Sohichi Hirose. "Mode-exciting method for Lamb wave-scattering analysis." *The Journal of the Acoustical Society of America* 115.3 (2004): 996-1005.
- Graff, Karl F. *Wave Motion in Elastic Solids*. Dover Publications, Inc., 1991. Print

Bi-orthogonality

- A set of functions $\{f_1(n,x), f_2(n,x)\}$ are bi-orthogonal if:

$$\int f_1(m,x) f_1(n,x) dx = C_1 \delta_{mn}$$

$$\int f_2(m,x) f_2(n,x) dx = C_2 \delta_{mn}$$

$$\int f_1(m,x) f_2(n,x) dx = 0$$

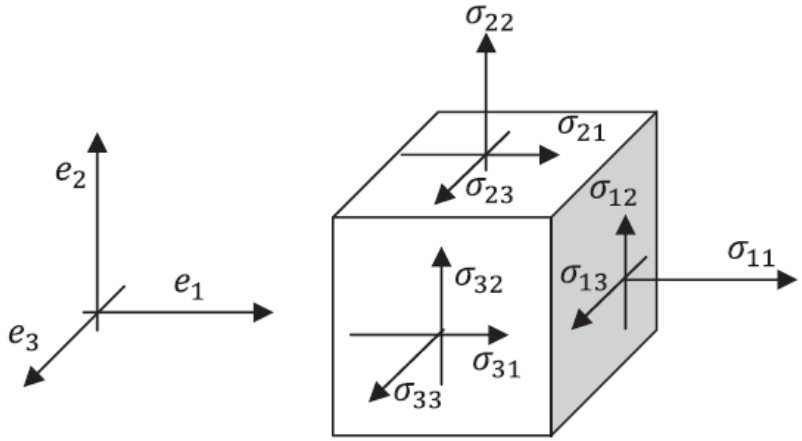
where $m \neq n$ and C_1 and C_2 are real constants.

Bi-orthogonality

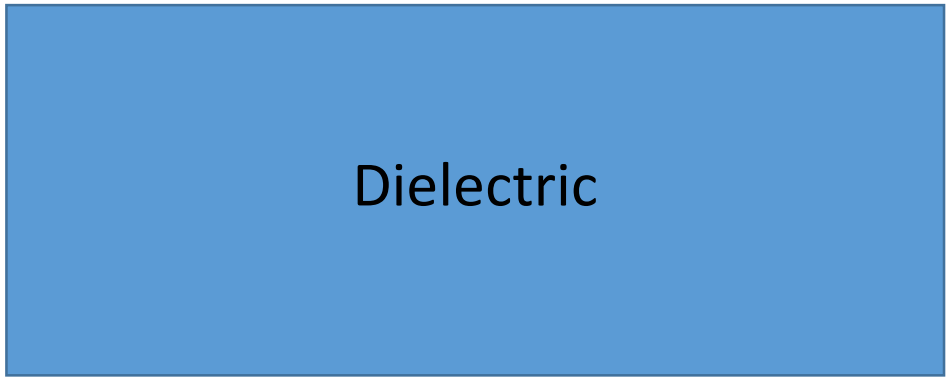
- In our specific case the bi-orthogonality relation is as follows:

$$\int \left(\sigma_{xz}^{*(1)} \, u_z^{(2)} - \, \sigma_{xx}^{(2)} \, u_x^{*(1)} \right) dz = 0$$

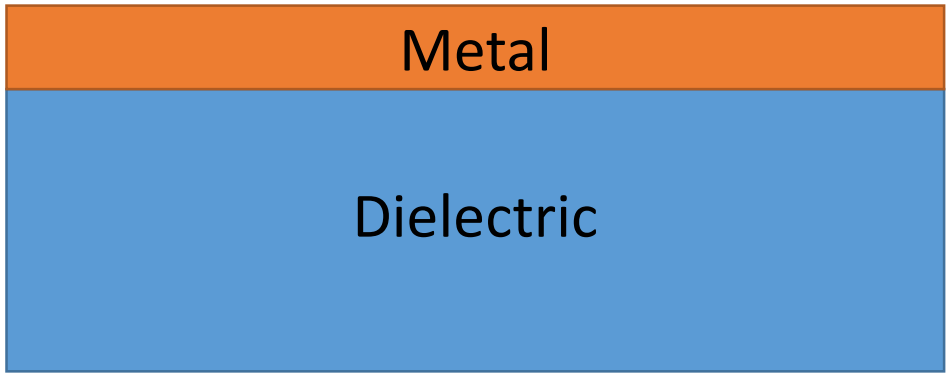
- σ and u are the stress and displacement in the plate
- superscripts represent eigenmodes
- subscripts refer to specific vector components
- asterisks signify complex conjugates



Simplified Problem

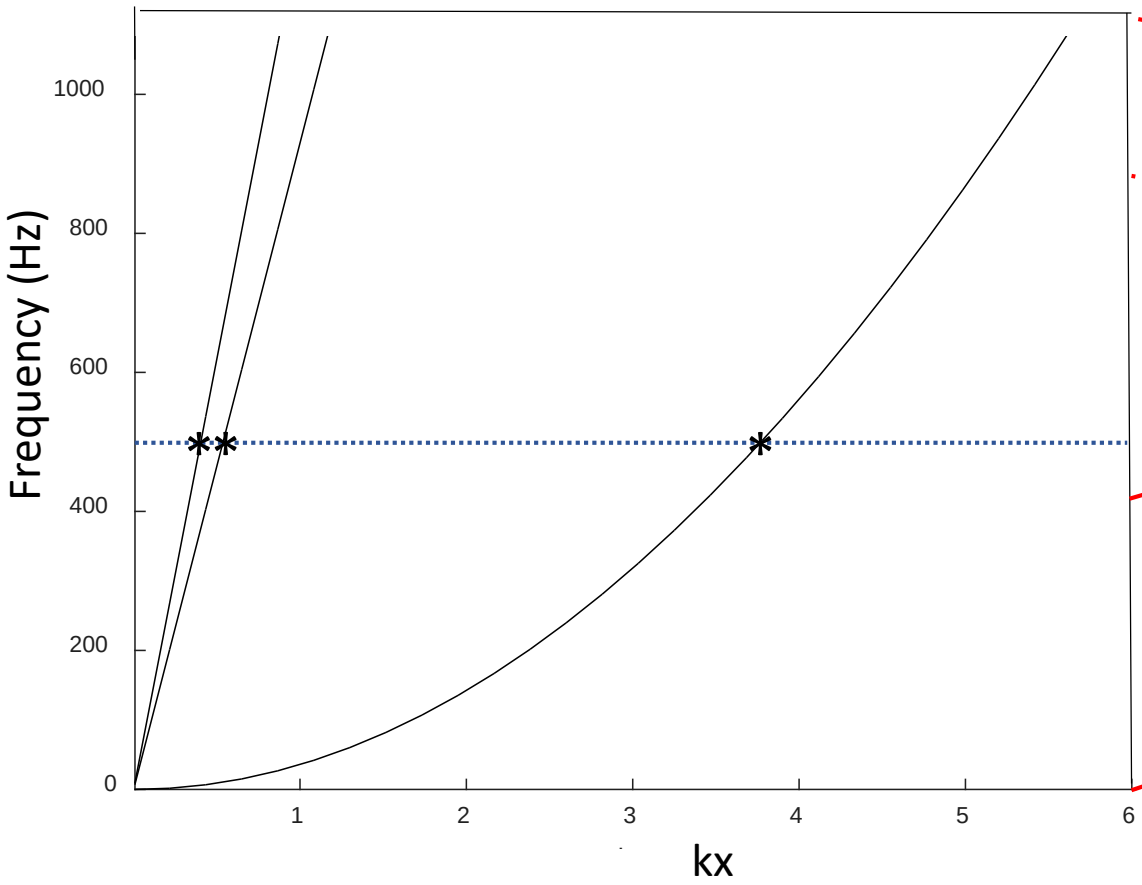


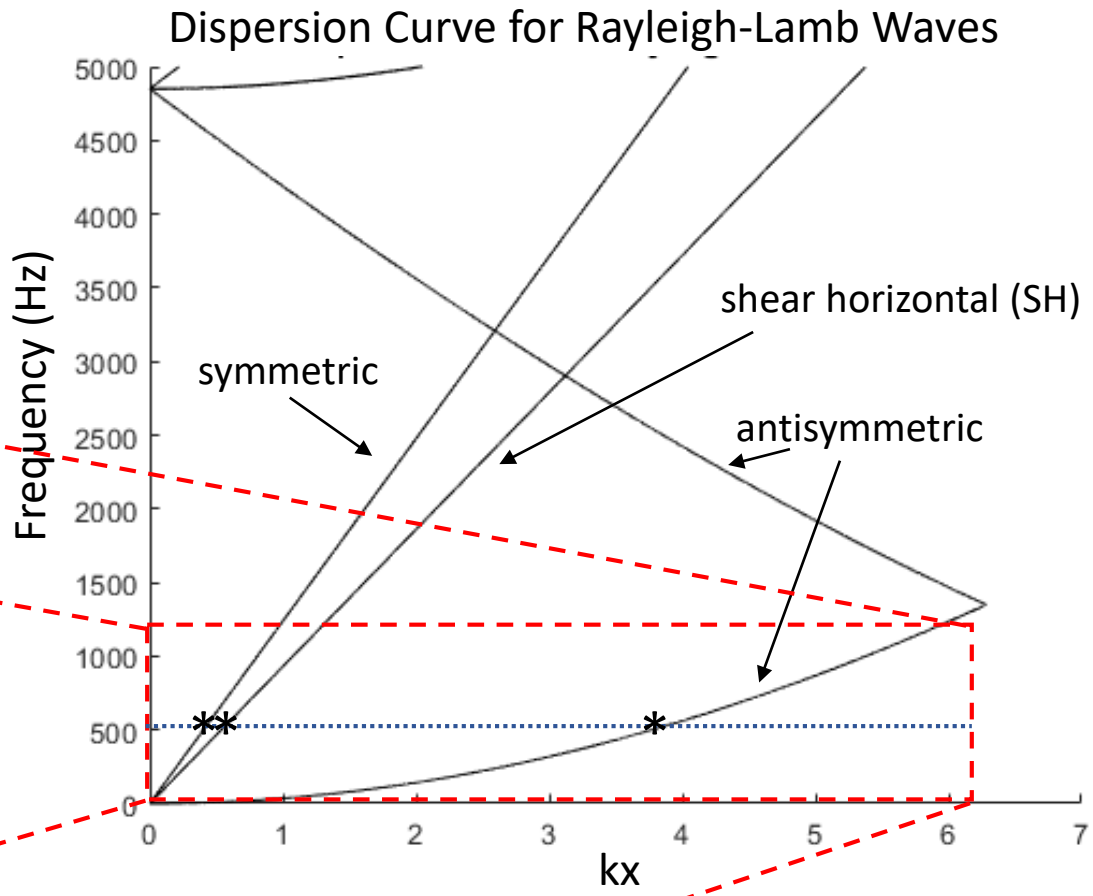
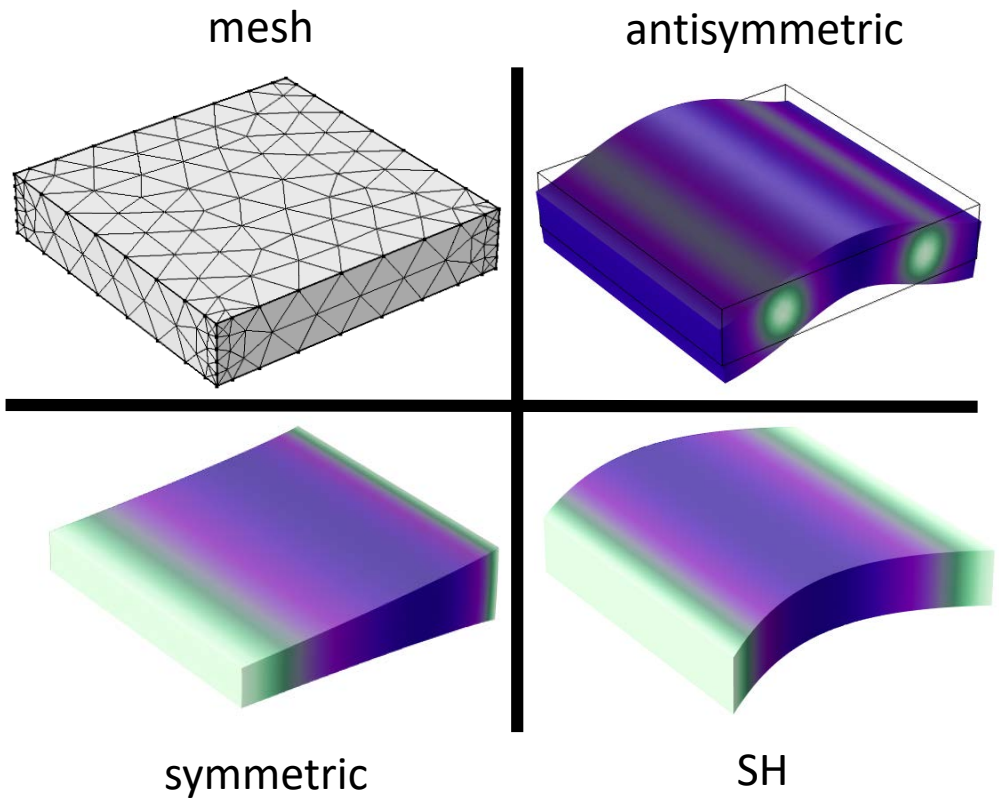
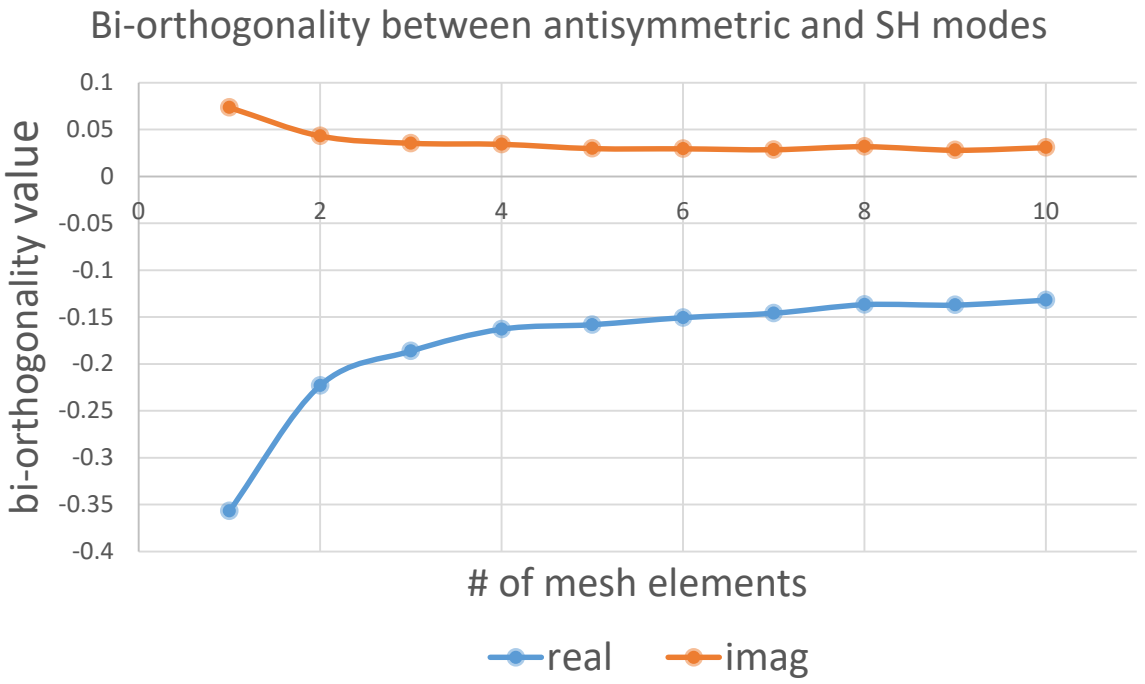
simplified version of plate



complete version of plate

Numerical Results





Numerical Results